A Progressive Error Estimation Framework for Photon Density Estimation

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Quiz
Quiz
Importance of Error Estimation

- ‘Guesswork’ does not work!
Importance of Error Estimation

- ‘Guesswork’ does not work!
- Key element for many applications
  - Predictive rendering (e.g., lighting engineering)
  - Error-driven computation
  - Theoretical error analysis
Definition of Error

- Difference between computed and exact

\[ E_i = L_i - L \]
Definition of Error

- Difference between computed and exact

\[ E_i = L_i - \boxed{L} \text{ Unknown} \]
Error in Unbiased Methods

- Path Tracing [Kajiya 86]

\[ E[E_i] \approx \sqrt{\text{Variance}_i} \]
Error in Biased Methods

- Photon Density Estimation [Jensen 96][Walter 98]

![Graph showing error vs. number of samples]

- Error vs. Number of Samples
- Bias (unknown)
Error in Biased Methods

- Progressive Photon Mapping [Hachisuka et al. 08]

Converging Bias (unknown)
Error in Biased Methods

- Error estimation in biased methods is challenging

\[ E_i = ? \]
Related Work
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• Variance-based estimation for unbiased methods
  [Lee et al. 85][Purgathofer 87][Tamstorf et al. 97]
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- Deterministic error bound in restricted cases
  [Ward et al. 88][Walter et al. 05]
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- Bias reduction
  [Myszkowski 97][Schregle 03]
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- Deterministic error bound in restricted cases
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- Bias reduction
  [Myszkowski 97][Schregle 03]

- Heuristic error estimation
  [Walter 98]
Contribution

Error estimator for photon density estimation
Method
Error in Biased Methods

- Difference between computed and exact

\[ E_i = L_i - L \]
Error in Biased Methods

- **Bias-Noise** decomposition

\[ E_i = L_i - L = B_i + N_i \]
Error in Biased Methods

- Bias-Noise decomposition

\[ E_i = L_i - L = B_i + N_i \]

Progressive Photon Mapping
Error Estimation in Monte Carlo Methods

• Can we estimate $E_i$?
Error Estimation in Monte Carlo Methods

• Can we estimate $E_i$?

Probably Not
Error Estimation in Monte Carlo Methods

\[ E_i = L_i - L \]
Error Estimation in Monte Carlo Methods

\[ L = L_i - E_i \]
Error Estimation in Monte Carlo Methods

\[ L = L_i - E_i \]

Estimating error is as difficult as estimating radiance
Can we estimate bounds of $E_i$?

$E_{\min,i} \leq E_i \leq E_{\max,i}$
Error Estimation in Monte Carlo Methods

- Can we estimate bounds of $E_i$?

$$E_{\text{min},i} \leq E_i \leq E_{\text{max},i}$$

Probably Not
Error Estimation in Monte Carlo Methods

- Integration of a rectangular function

\[ \int F(x) \, dx = 1 \]
Error Estimation in Monte Carlo Methods

- Integration of a rectangular function
Error Estimation in Monte Carlo Methods

- Integration of a rectangular function
Error Estimation in Monte Carlo Methods

- Integration of a rectangular function

\[ \int F(x) \, dx = 1 \]
Error Estimation in Monte Carlo Methods

- Integration of a rectangular function
Error Estimation in Monte Carlo Methods

- Integration of a rectangular function

Error is not bounded
Error Estimation in Monte Carlo Methods

Probability\left(E_{\text{min},i} \leq E_i \leq E_{\text{max},i}\right) = 90\%

Drop a few cases where

\[ E_{\text{min},i} \leq E_i \leq E_{\text{max},i} \]

is false
Stochastic Error Bounds

- Bounds that are true with some probability
- Well-known concept in computational statistics

\[
P\left(\min_{i} E_{\min,i} \leq E_{i} \leq \max_{i} E_{\max,i}\right) = 1 - \beta
\]

Stochastic error bound

User-defined Confidence
Stochastic Error Bound Derivation

\[ L_i - L = E_i = B_i + N_i \]
Stochastic Error Bound Derivation

• Subtract bias

\[ L_i - L - B_i = E_i - B_i = N_i \]
Stochastic Error Bound Derivation

- Noise follows the t-distribution

\[ L_i - L - B_i = E_i - B_i = N_i \]

\[ P(-N_b \leq N_i \leq N_b) = 1 - \beta \]

\[ N_b = C_i,1 - \frac{\beta}{2} \sqrt{\text{Variance}_i} \]
Stochastic Error Bound Derivation

- Add back bias

\[ L_i - L = E_i = B_i + N_i \]

\[ P(-N_b + B_i \leq E_i \leq N_b + B_i) = 1 - \beta \]

\[ N_b = C_i, 1 - \frac{\beta}{2} \sqrt{\text{Variance}_i} \]
Stochastic Error Bound Derivation

- Take the absolute value

\[ L_i - L = E_i = B_i + N_i \]

\[ P(|E_i| \leq |N_b| + |B_i|) \leq 1 - \beta \]

\[ N_b = C_{i,1-\beta} \sqrt{\frac{\text{Variance}}{i}} \]
Stochastic Error Bound Derivation

\[ L_i - L = E_i = B_i + N_i \]

Stochastic error bound

\[
P(\left| E_i \right| \leq E_{b,i}) \leq 1 - \beta
\]

User-defined Probability

\[
E_{b,i} = C_i, 1 - \beta \sqrt{\frac{\text{Variance}}{i}} + \left| B_i \right|
\]
Stochastic Error Bound Derivation

\[ L_i - L = E_i = B_i + N_i \]

\[ P(|E_i| \leq E_{b,i}) \leq 1 - \beta \]

\[ E_{b,i} = C_i, 1 - \frac{\beta}{2} \sqrt{\frac{\text{Variance}}{i}} + |B_i| \]

Error due to Noise
Stochastic Error Bound Derivation

\[ L_i - L = E_i = B_i + N_i \]

\[ P(|E_i| \leq E_{b,i}) \leq 1 - \beta \]

\[ E_{b,i} = C_{i,1-rac{\beta}{2}}\sqrt{\text{Variance}_i} + |B_i| \]

Error due to Bias
Challenges

\[ E_{b,i} = C_{i,1} - \frac{\beta}{2} \sqrt{\text{Variance}_i} + |B_i| \]

- \( B_i \) (bias) is unknown
Challenges

\[ E_{b,i} = C_{i,1 - \frac{\beta}{2}} \sqrt{\frac{\text{Variance}}{i}} + |B_i| \]

- \( B_i \) (bias) is unknown
- Variance estimation assumes i.i.d.
  - independent and identically distributed
  - not true in progressive photon mapping
Bias Estimation

- $B_i$ (bias) is unknown
- Well-known approximation [Silverman 86]

$$B_i \approx k_2 R_i^2 \Delta L$$

- $k_2$ constant
- $R_i$ search radius
- $\Delta L$ Laplacian of radiance
Bias Estimation

- $B_i$ (bias) is unknown
- Well-known approximation [Silverman 86]

$$B_i \approx k_2 R_i^2 \Delta L$$

- $k_2$ constant
- $R_i$ search radius
- $\Delta L$ Laplacian of radiance
- Unknown
Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

\[
L_i(x) = \sum K(x_p - x) f_r(x, \omega, \omega_p) \Phi(x_p, x) \frac{\pi R_i^2}{\pi R_i^2}
\]

Kernel
Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

\[
\Delta L_i(x) = \sum \frac{\Delta K(x_p - x) f_r(x, \omega, \omega_p) \Phi(x_p, x)}{\pi R_i^2}
\]
Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

\[
\Delta L_i(x) = \sum \frac{\Delta K(x_p - x)f_r(x, \omega, \omega_p)\Phi(x_p, x)}{\pi R_i^2}
\]

Extended to progressive photon mapping
Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

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\Delta L_i(x) = \sum \frac{\Delta K(x_p - x) f_r(x, \omega, \omega_p) \Phi(x_p, x)}{\pi R_i^2}
\]

Laplacian of the kernel

Extended to progressive photon mapping

- Applicable to any order
Radiance Derivative Estimation

- Kernel-based estimation of Laplacian

\[ \Delta L_i(x) = \sum \frac{\Delta K(x_p - x)f_r(x, \omega, \omega_p)\Phi(x_p, x)}{\pi R_i^2} \]

Laplacian of the kernel

Extended to progressive photon mapping
- Applicable to any order
- Convergent
Variance Estimation

- Variance estimation assumes i.i.d.
- Not true in progressive photon mapping
Variance Estimation

- Two key observations
- Photon tracing itself is independent
- Dependency is only in radius reduction
Variance Estimation

- Two key observations
- Photon tracing itself is independent
- Dependency is only in radius reduction

Bias-corrected radiance

\[ L_i' = L_i - B_i \]

\[ \text{Variance} \approx \frac{\sum L_i'^2 - \left( \sum \frac{L_i'}{i} \right)^2}{i - 1} \]
Key Points

- Approximate stochastic error bounds
- Convergent derivative estimator
- Bias/Noise estimators valid for PPM
Results
Experiments Setup

- Progressive Photon Mapping [Hachisuka et al. 08]
- 15k photons per pass
- Three test scenes with full global illumination
Calculated Probability of Bounds

\[ P(\left| E_i \right| \leq E_{b,i}) \leq 1 - \beta \]

within 5% deviation

1 - \beta = 50%

1 - \beta = 90%
Bounded Pixel Visualization

**Bounded/Not bounded**

\[ 1 - \beta = 50\% \]  \hspace{1cm}  \[ 1 - \beta = 90\% \]
Noise-Bias Ratio
Automatic Rendering Termination

Stochastic Bound Per Pixel (50%)

\[ P(\left| E_i \right| \leq E_{b,i}) \leq 50\% \]

Stop rendering if

\[ \text{Average}[E_{b,i}] < E_{\text{thr}} \]

User-specified allowable error
Automatic Rendering Termination

specified: 0.25
actual: 0.1916

specified: 0.125
actual: 0.09294

specified: 0.0625
actual: 0.04482

1.3 times overestimation on average
Automatic Rendering Termination

Estimated Bound
Actual Error
Future Work

- Various applications of error estimates
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- Various applications of error estimates
- Better stopping criterion
Future Work

- Various applications of error estimates
- Better stopping criterion
- Error-driven adaptive sampling
Future Work

- Various applications of error estimates
- Better stopping criterion
- Error-driven adaptive sampling
- Optimal search radius based on error
Future Work

- Various applications of error estimates
- Better stopping criterion
- Error-driven adaptive sampling
- Optimal search radius based on error
- ... and many more
Future Work

• Various applications of error estimates
• Better stopping criterion
• Error-driven adaptive sampling
• Optimal search radius based on error
• ... and many more
• Extension to stochastic PPM [Hachisuka et al. 09]
Future Work

- Various applications of error estimates
- Better stopping criterion
- Error-driven adaptive sampling
- Optimal search radius based on error
- ... and many more
- Extension to stochastic PPM [Hachisuka et al. 09]
- More accurate bias estimation
Conclusion

- Error estimation for photon density estimation
- General and non-heuristic (gives an error-bar)
- Estimator applies to progressive photon mapping
Conclusion

- Error estimation for photon density estimation
- General and non-heuristic (gives an error-bar)
- Estimator applies to progressive photon mapping

Take-home message:

First step toward answering: “How many photons are enough?”
Acknowledgements

- ATI fellowship 2008-2009
- Youichi Kimura (Studio Azurite): modeling
- Matus Telgarsky, Daniel Hsu (UCSD): discussion
Thank You