Extended Path Integral Formulation for Volumetric Transport

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[Jensen and Christensen 1998]

[Pauly et al. 2000]

[Křivánek et al. 2014]

[Jarosz et al. 2011]
Bidirectional path tracing [Pauly et al. 2000]
Volume photon mapping [Jensen and Christensen 1998]
Beam radiance estimate [Jarosz et al. 2008]
Photon beams [Jarosz et al. 2011]
Comprehensive theory [Jarosz et al. 2011]
Comprehensive theory [Jarosz et al. 2011]

3D blur

2D blur

1D blur
UPBP formulation

- Unified points, beams, and paths as sampling techniques for volumes

[Křivánek et al. 2014]
Dimensionality of paths

Path integral: **Four** vertices

Density estimation: **Five** vertices

**Same path length**
Merge vertices
Consider all the paths which result in the same merged path
Accept according to the probability of merging

\[
Probr \left[ x \equiv y \right] = \int p(y') dy'
\]
\[ \text{Prob} \left[ x \equiv y \right] = \int p(y') \, dy' \]
$Prob[x \equiv y] = \int p(y') \, dy'$
UPBP formulation

• Three steps to match with BDPT
  1. Merge subpaths
  2. Consider all the paths which result in the same merged path
  3. Accept the path with the probability of merging

Beam-Beam 1D

Corresponds to contraction of density estimation path space

$$\text{Prob}[x \equiv y] = \int p(y') dy'$$
UPS/VCM formulation

- Unified path integration and photon density estimation for surfaces

[Hachisuka et al. 2012]  [Georgiev et al. 2012]
Vertex Connection and Merging

- **Contract** the space of density estimation into the original path space
Vertex Connection and Merging

- **Contract** the space of density estimation into the original path space

Path integration

Vertex merging
Unified Path Sampling

- **Extend** the original path space to include photon density estimation
Unified Path Sampling

- **Extend** the original path space to include photon density estimation
Differences

- **VCM**: precise for path integration, approximate for density estimation
- **UPS**: precise for density estimation, approximate for path integration
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Path integral formulation

Vertices are fully connected
Extended path integral formulation

Allow disconnected vertices
Extended path integral formulation

Blurring kernel as throughput of disconnected vertices

\[ K_{3D}(x, y) \]
Point-Point 3D

Precisely models photon density estimation

\[ K_{3D}(x, y) \]
3D blur to 2D blur

\[ K_{3D}(x, y) \]
3D blur to 2D blur

\[ K_{2D}(x, y) = K_{3D}(x, y) \delta(x_t - t_K) \]

Flatten a sphere into a disc
Beam-Point 2D

\[ K_{2D}(x, y) \]
Beam-Point 2D

$K_{2D}(x, y)$

Beam-point 2D = deterministic sampling of one distance
2D blur to 1D blur

\[ K_{2D}(x, y) \]
2D blur to 1D blur

\[ K_{1D}(x, y) = K_{2D}(x, y) \delta(x_t - t'_K) \]

Flatten a disc into a line
Beam-Beam 1D

\[ K_{1D}(x, y) \]
Beam-beam 1D = deterministic sampling of two distances

$K_{1D}(x, y) = K_{2D}(x, y)$
Integral over the intersection interval

\[ \int_{\text{intersection}} f(x, y) K(x, y) \, dy \]
Stochastic sampling within the interval

$p(t_y)$
Beam-Beam 2D

\[ p(t_y) \]

\[ \delta(t_x - t(y_{proj})) \]
Beam Beam 2D

\[ K_{1D}(x, y) = K_{2D}(x, y) \]
Beam-Beam 3D
Beam-Beam 3D [Jarosz et al. 2011]

Double integral over the intersection intervals (usually intractable)

\[ \int \int f(x, y)K(x, y)\, dt_y \, dt_x \]
Beam-Beam 3D

\[ p(t_y) \]
Beam-Beam 3D

\[ p(t_y) \]
Beam-Beam 3D

$K_{3D}(x, y) = K_2D(x, y)$

Same 3D kernel as point-point 3D
Beam-Beam 3D

Simple Monte Carlo path sampling (no longer intractable)
Beam-Point 3D
Beam-Point 3D

Same 3D kernel as point-point 3D
Bidirectional path tracing
Bidirectional path tracing

\[ p(y) = \delta(x - y) \]

Duplicate a vertex
Bidirectional path tracing

\[ p(y) = \delta(x - y) \]

\[ K_{3D}(x, y) = \delta(x - y) \]

Delta kernel leads to the original path integral formulation
Biased bidirectional path tracing

\[ p(y) \neq \delta(x - y) \]

\[ K_{3D}(x, y) \neq \delta(x - y) \]

Take disconnected vertices via blurring kernel
Virtual perturbation

\[ p(y) \approx \delta(x - y) \]

\[ K_{3D}(x, y) \neq \delta(x - y) \]

Approximate the implementation of biased BDPT by regular BDPT
Conclusion

• Extension of the path space for volumetric light transport
• Better explains density estimation compared to merging
• Formulate beam as Monte Carlo distance sampling
• Enables a practical beam-beam 3D estimator

Fills a theoretical gap in the unified formulation for volumes